The Longest Perpetual Reductions in Orthogonal Expression Reduction Systems

Zurab Khasidashvili
School of Information Systems, UEA
Norwich NR4 7TJ England
zurab@sys.uea.ac.uk

Abstract

We consider reductions in Orthogonal Expression Reduction Systems (OERS), that is, Orthogonal Term Rewriting Systems with bound variables and substitutions, as in the λ-calculus. We design a strategy that for any given term t constructs a longest reduction starting from t if t is strongly normalizable, and constructs an infinite reduction otherwise. The Conservation Theorem for OERSs follows easily from the properties of the strategy. We develop a method for computing the length of a longest reduction starting from a strongly normalizable term. We study properties of pure substitutions and several kinds of similarity of redexes. We apply these results to construct an algorithm for computing lengths of longest reductions in *strongly persistent* OERSs that does not require actual transformation of the input term. As a corollary, we have an algorithm for computing lengths of longest developments in OERSs.

1 Introduction

A strategy is *perpetual* if, given a term t, it constructs an infinite reduction starting from t whenever such a reduction exists, that is, whenever t is not strongly normalizable. Perpetual strategies are mostly interesting because termination of a perpetual reduction (constructed according to a perpetual strategy) implies strong normalization of the initial term. For orthogonal (left-linear and non-overlapping) TRSs a very simple perpetual strategy exists — just contract any innermost redex (O'Donnell [15]). In fact, any *complete* strategy, i.e., a strategy that in each term contracts a redex that does not erase any other redex, is perpetual. Moreover, one can even reduce redexes all erased arguments of which are strongly normalizable (Klop [11]).

It is easy to see that in any infinite reduction a redex that itself has an infinite reduction, call it an *infinite* redex, is contracted. Thus in order to construct an infinite reduction one should try to retain at least one *potentially infinite* redex — a subterm that can become an infinite redex (more precisely, that has a descendant under some reduction that is an infinite redex). Thus any strategy that does not erase potentially infinite redexes is perpetual. In OTRSs, any potentially infinite redex necessarily has an infinite reduction. That is why all the above strategies are perpetual. In orthogonal Expression Reduction Systems (that is, TRSs with bound variables and substitution mechanism [6]) a strongly normalizable subterm may also be a potentially infinite redex — after contraction of an outer redex a term can be substituted in it that makes the subterm no longer strongly normalizable. Thus innermost reductions and complete reductions are no longer perpetual in OERSs. Therefore one can erase only strongly normalizable arguments in which no substitution of external subterms is possible. For the lambda-calculus, such a strategy was found by Barendregt et al. [2].

In this paper, we design a perpetual strategy for all OERSs. It is a generalization of the perpetual strategy of the λ-calculus and of the *limit* strategy [8]. Our aim is not only to construct an infinite reduction of any given term t whenever it exists, but also to construct a longest possible one if t is strongly normalizable and to find a method to characterize the lengths of longest reductions. The idea is that, as mentioned abov